PHYS 481: Quantum Mechanics

Assignment 03 - Will St. John

Problem 3

In[1]:=

```
(*Define Ψ and and ψ*)
Ψ = Sqrt[105/L^7] x^2 (L - x) (*Wavefunction from x=0 to x=L*);
ψ = Sqrt[2/L] Sin[π n/L x] (*Groundstate energy of particle in infinite square well*);
```

$$\Psi = \sqrt{\frac{105}{L^7}}\, x^2 (L - x)$$

$$\psi = \sqrt{\frac{2}{L}}\, \mathrm{Sin}\left[\frac{\pi n}{L} x\right]$$

(a) What is the probability that a measurement of the energy of the particle yields the ground-state energy?

In[3]:=

```
Simplify[Integrate[(Sqrt[105/L^7] x^2 (L - x))^2, {x, 0, L}]]
(*Check that wavefunction is normalized*)

Simplify[(Integrate[ψ Ψ, {x, 0, L}])^2 /. {n → 1}] // N
(*Calculate the probability of a energy measurement being in the ground state*)
```

Out[3]= 1

Out[4]= 0.873736

Problem 5

In[5]:=

```
(*Define the Rodrigues Formula, recursion relation, and generating function*)
H[n_] := (-1)^n e^(ξ^2) D[e^(-ξ^2), {ξ, n}];
H2[n_] := 2 ξ H[n] - 2 n H[n - 1];
Gen[n_] := D[e^(-z^2+2 z ξ), {z, n}];
```

(a) Use the Rodrigues formula to derive H3 and H4

In[8]:=
```
Simplify[H[3]]
Simplify[H[4]]
```

Out[8]= $4\,\xi\,\left(-3+2\,\xi^2\right)$

Out[9]= $4\,\left(3-12\,\xi^2+4\,\xi^4\right)$

(b) Use the recursion relation to obtain H5 and H6

In[10]:=
```
Simplify[H[5]] (*H5 from Rodrigues*)
Simplify[H2[4]] (*H5 from recursion*)

Simplify[H[6]] (*H6 from Rodrigues*)
Simplify[H2[5]] (*H6 from recursion*)
```

Out[10]= $8\,\xi\,\left(15-20\,\xi^2+4\,\xi^4\right)$

Out[11]= $8\,\xi\,\left(15-20\,\xi^2+4\,\xi^4\right)$

Out[12]= $8\,\left(-15+90\,\xi^2-60\,\xi^4+8\,\xi^6\right)$

Out[13]= $8\,\left(-15+90\,\xi^2-60\,\xi^4+8\,\xi^6\right)$

(c) Differentiate H5 and H5

In[14]:=
```
Simplify[D[H[5], ξ]] (*Differentiate H5*)
Simplify[2*5 * H[4]] (*RHS of Differential Hermite polynomial*)

Simplify[D[H[6], ξ]] (*Differentiate H6*)
Simplify[2*6 * H[5]] (*RHS of Differential Hermite polynomial*)
```

Out[14]= $40\,\left(3-12\,\xi^2+4\,\xi^4\right)$

Out[15]= $40\,\left(3-12\,\xi^2+4\,\xi^4\right)$

Out[16]= $96\,\xi\,\left(15-20\,\xi^2+4\,\xi^4\right)$

Out[17]= $96\,\xi\,\left(15-20\,\xi^2+4\,\xi^4\right)$

(d) Obtain H1, H2, H3 from the generating function/taylor series expansion

In[18]:=
```
Simplify[H[1]] (*Calculate H1 from Rodrigues formula*)
Simplify[Gen[1] /. (z → 0)] (*Calculate H1 from derivative of generating function*)

Simplify[H[2]] (*Calculate H2 from Rodrigues formula*)
Simplify[Gen[2] /. (z → 0)] (*Calculate H2 from derivative of generating function*)

Simplify[H[3]] (*Calculate H3 from Rodrigues formula*)
Simplify[Gen[3] /. (z → 0)] (*Calculate H3 from derivative of generating function*)
```

Out[18]=

$2\,\xi$

Out[19]=

$2\,\xi\,\mathrm{Log}[e]$

Out[20]=

$-2+4\,\xi^2$

Out[21]=

$2\,\mathrm{Log}[e]\,\left(-1+2\,\xi^2\,\mathrm{Log}[e]\right)$

Out[22]=

$4\,\xi\,\left(-3+2\,\xi^2\right)$

Out[23]=

$4\,\xi\,\mathrm{Log}[e]^2\,\left(-3+2\,\xi^2\,\mathrm{Log}[e]\right)$